Краткие сообщения ОИЯИ №3[66]-94 *JINR Rapid Communications No.3[66]-94*
УДК 539.142

NUCLEON CORRELATION EFFECTS ON Y-SCALING QUANTITIES IN NUCLEI*

M.K.Gaidarov[1], A.N.Antonov[1], S.S.Dimitrova[1], M.V.Stoitsov[1]

The asymptotic scaling function $F(y)$ and the binding correction $B(y)$ as well as the mean kinetic and removal energies are calculated in the cases of the ^{4}He, ^{12}C, ^{16}O and ^{40}Ca nuclei using the nucleon momentum distributions obtained within the Jastrow correlation method and the phenomenological model accounted for short-range and tensor nucleon-nucleon correlations. The scaling functions $F(y)$ differ from those obtained in the mean-field approximation and are in qualitative agreement with the available experimental data. It is shown that the binding correction $B(y)$ can be explicitly evaluated using a realistic nuclear spectral function. The account for the nucleon-nucleon correlations gives increased values of the mean kinetic $\langle T \rangle$ and mean removal $\langle E \rangle$ energy (in comparison with their values in the mean-field approximation) and leads to correct values of the binding energy per nucleon in the nuclei considered.

The investigation has been performed at INRNE (Bulgaria) in collaboration with LTP, JINR.

Нуклонные корреляционные эффекты на Y-скейлинговых величинах в ядрах

М.К.Гайдаров, А.Н.Антонов, С.С.Димитрова, М.В.Стоицов

Асимптотическая скейлинговая функция $F(y)$, поправка на энергию связи $B(y)$, а также средние кинетическая и энергия связи вычислены для ядер ^{4}He, ^{12}C, ^{16}O и ^{40}Ca с использованием нуклонных импульсных распределений, полученных в рамках метода ястровских корреляций и феноменологической модели, учитывающей короткодействующие и тензорные нуклон-нуклонные корреляции. Скейлинговые функции $F(y)$ отличаются от полученных в приближении среднего поля и находятся в качественном согласии с имеющимися экспериментальными данными. Показано, что поправка на связь $B(y)$ точно оценена с использованием реалистичной ядерной спектральной функции. Учет нуклон-нуклонных корреляций дает увеличенные значения средней кинетической энергии $\langle T \rangle$ и средней энергии связи $\langle E \rangle$ по сравнению с их значениями в приближении среднего поля и ведут к правильным значениям энергии связи на нуклон для рассмотренных ядер.

Работа выполнена в ИЯИЯЭ (Болгария) в сотрудничестве с ЛТФ, ОИЯИ.

---

*This work is partially supported by the Bulgarian National Science Foundation under the Contract No.Ф-32.

[1]Permanent address: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria.

1. Introduction

Significant part of the recent nuclear experiments, both in high and low energies, poses the question about the limits of the mean-field approximation (MFA) in nuclear theory. We shall mention the deep-inelastic proton-, inclusive and exclusive electron-scattering in nuclei which show the existence of high-momentum components in the nucleon momentum distribution as well as partial depletion of the levels below the Fermi level and partial filling above it in the nuclear ground state [1]. These results are in contradiction with the predictions of the shell model. The reason of this are the effects of short-range and tensor nucleon-nucleon correlations in nuclei which are related to specific peculiarities of the nucleon-nucleon forces at small distances. This imposes the development of correlated methods in nuclear theory which are going beyond the limits of the MFA.

A plausible method for studying of short-range correlation (SRC) effects in nuclei is the y-scaling method [2—10]. Since West's pioneer work [2], there has been a growth of interest in y-scaling analysis, both in its experimental and theoretical aspects. This is motivated by the importance of extracting nucleon momentum distributions from the experimental data. Furthemore, the y-scaling method enables us to see how the characteristics of the system considered at finite momentum transfer q differ from those obtained in the framework of the Plane Wave Impulse Approximation (PWIA). From experimental point of view, the possibility of extracting the nucleon momentum distribution from the inclusive electron quasielastic scattering data relies on the knowledge of the scaling function in the asymptotic limit and there are no clear criteria to decide whether the available data which are necessarily obtained for large but finite values of q can be associated with those from the analysis of the true asymptotic region. It has been shown that if a proper theory of y-scaling (taking into account the nucleon binding and momentum) is adopted, then the extracted nucleon momentum distributions are in good agreement with those obtained in a more direct way from the exclusive electron scattering $(e, e'p)$ experiments. This fact confirms the expectation that the corresponding asymptotic scaling function should agree with the experimental one even if the experimental data are affected by the final state interaction (FSI).

A detailed study of the momentum distribution and its relation with the spectral function shows that it can be divided into two parts corresponding to low- and high-momentum components. These components in the nucleus with A nucleons are associated with the ground state and high virtual excitations of the spectator system with $A-1$ nucleons. Such investigation allows one to calculate the scaling function and some important nuclear characteristics.

In this paper we connect the results on the SRC effects obtained in the Jastrow correlation method (JCM) [11—13] and in the phenomenological method (PM) accounting for short-range and tensor correlations from [14,15] with the quantities which are analysed in the y-scaling method. The aim of our work is, using the nucleon momentum distributions obtained in the correlation methods mentioned above, to calculate the scaling function, the binding correction function as well as the mean kinetic and mean removal energies in ^{4}He, ^{12}C, ^{16}O and ^{40}Ca nuclei. The results for the asymptotic function are compared with the available experimental data for the cases of ^{4}He and ^{12}C nuclei. This comparison serves as a test for the correlation methods. The results for the ^{16}O and ^{40}Ca nuclei are predictions of the JCM and PM concerning the y-scaling quantities. The calculations of the binding correction function and the mean kinetic and removal energies give an additional information on the extent to which the SRC are accounted for in the correlation methods considered in the present work and on their reliability analyzing quantities which are sensitive to the SRC.

The definitions and discussion of the main y-scaling quantities, as well as the basic relationships of the correlation methods considered in this work are given in Section 2 of the paper. The results for the asymptotic scaling function, for the binding correction function, as well as for the mean kinetic and mean removal energies are presented in Section 3. The concluding remarks are given in Section 4.

2. Theoretical Basis

2.1. The Y-Scaling Method

The general concepts of the y-scaling method for the description of the electron quasielastic scattering by nuclei have been introduced by West [2]. Using the scaling variable y as a kinematical variable, the inclusive cross section σ_2 can be presented in the PWIA in the form [10]:

$$\sigma_2 = (Z\overline{\sigma_{\mathrm{ep}}} + N\overline{\sigma_{\mathrm{en}}}) \overline{\left| \frac{\partial\omega}{k\partial \cos\alpha} \right|^{-1}} F(q, y), \tag{1}$$

where $\overline{\sigma_{\mathrm{ep}(n)}}$ is the relativistic electron-proton (neutron) cross section in the scattering of an electron by an off-shell nucleon with momentum $\mathbf{k}$, $\omega(q)$ is the energy transfer, q is the momentum transfer and $\left|\frac{\partial\omega}{k\partial\cos\alpha}\right|^{-1}$ is the kinematical factor. The nuclear structure function

$$F(q, y) = 2\pi \int_{E_{min}}^{E_{max}(q,y)} dE \int_{k_{min}(q,y,E)}^{k_{max}(q,y,E)} P(k, E)\, kdk \qquad (2)$$

is a function of q and the scaling variable

$$y = (\mathbf{kq})/q = M(\omega - q^2/2M)/q. \qquad (3)$$

In (2) $P(k, E)$ is the spectral function, the limits of integration are determined by the energy conservation, $E_{min} = |E_A| - |E_{A-1}|$, E_A and E_{A-1} being the ground state energies of the A and $A - 1$ nuclei, respectively. The scaling variable y satisfies the equation:

$$\omega + M_A = [M^2 + (q + y)^2]^{1/2} + [M^2_{A-1} + y^2]^{1/2}, \qquad (4)$$

where M_A and M_{A-1} are the masses of the A and $A - 1$ nuclei in their ground state. y has a meaning of a minimal longitudinal (along q) momentum of a nucleon with the minimal value ($E_{min} = M + M_{A+1} - M_A$, M being the proton mass) of the separation energy, i.e., $|y| = k_{min}(E_{min})$. The spectral function in (2) is usually given in the general form:

$$P(k, E) = P_{gr}(k, E) + P_{ex}(k, E), \qquad (5)$$

where

$$P_{gr}(k, E) = n_{gr}(k)\,\delta\,(E - E_{min}) \qquad (6)$$

is the probability distribution that the final ($A - 1$) system is left in the ground state (corresponding to the excitation energy $E^*_{A-1} = 0$ and $E = E_{min}$), whereas $P_{ex}(k, E)$ is probability distribution that the final ($A - 1$) system is left in any of its excited states (with $E^*_{A-1} \neq 0, E = E_{min} + E^*_{A-1}$). The relation between the spectral function and the momentum distribution is

$$n(k) = \int_{E_{min}}^{\infty} P(k, E)\, dE = n_{gr}(k) + \int_{E_{min}}^{\infty} P_{ex}(k, E)\, dE = n_{gr}(k) + n_{ex}(k). \qquad (7)$$

Indeed, the momentum distribution can be presented as follows:

$$n(\mathbf{k}) = |\langle\Psi_0^{(A-1)}|\hat{\Psi}(\mathbf{k})|\Psi_0^{(A)}\rangle|^2 + \sum_{f\neq 0} |\langle\Psi_f^{(A-1)}|\hat{\Psi}(\mathbf{k})|\Psi_0^{(A)}\rangle|^2, \quad (8)$$

where $\hat{\Psi}(\mathbf{k})$ is the annihilation operator of a nucleon with momentum $\mathbf{k}$, $\Psi_0^{(A)}$ is the wave function of the ground state of the A-nucleon system, $\Psi_0^{(A-1)}$ and $\Psi_f^{(A-1)}$ are the wave functions of the $(A-1)$-nucleon system in its ground and excited (f) state, respectively. It can be seen that the first term in (8) reproduces $n_{gr}(k)$, whereas the second one gives $n_{ex}(k)$ from (7).

The separation of the spectral function (5) has been adopted (e.g., [6,10]) in order to single out the nucleon binding effect (coming from $P_{ex}(k, E)$) on the scaling function. We use model spectral functions in which only average excitation energy of the final nuclear system is considered:

$$P_{ex}(k, E) = n_{ex}(k)\,\delta(E - \overline{E_{ex}}). \quad (9)$$

The value of $\overline{E_{ex}}$ can be calculated from the energy weighed sum rule [16]:

$$\frac{E_A}{A} \equiv |\varepsilon_A| = \frac{1}{2}\{\langle E\rangle - \langle T\rangle (A-2)/(A-1)\}, \quad (10)$$

where

$$\langle T\rangle = \int (k^2/2M)\, P(k, E)\, d\mathbf{k}\, dE \quad (11)$$

and

$$\langle E\rangle = \int E P(k, E)\, d\mathbf{k}\, dE \quad (12)$$

are the mean kinetic and mean removal energies, respectively, and $|\varepsilon_A|$ is the binding energy per particle. It follows from (5), (6), (9) and (12) that

$$\langle E\rangle = E_{min} S_{gr} + \overline{E_{ex}} S_{ex}, \quad (13)$$

where

$$S_{gr} = 4\pi \int n_{gr}(k)\, k^2 dk \quad (14)$$

and

$$S_{ex} = 4\pi \int n_{ex}(k)\, k^2 dk \quad (15)$$

are the occupation probabilities. The normalization of the spectral function is

$$4\pi \int P(k, E)\, k^2\, dk dE = 1. \quad (16)$$

It was shown in [8] that due to the behaviour of the spectral function $P(k, E)$ at large k and E and using eq. (5), the structure function can be presented in the form:

$$F(q, y) = 2\pi \int_{|y|}^{\infty} n_{gr}(k)\, kdk + 2\pi \int_{E_{min}}^{\infty} dE \int_{k_{min}(q,y,E)}^{\infty} P_{ex}(k, E)\, kdk, \qquad (17)$$

where the first term scales in y, but the second term represents a «scaling violation» (k_{min} depends on q) due to the nucleon binding. In the asymptotic limit ($q \to \infty$)

$$\lim_{q\to\infty} k_{min}(q, y, E) \equiv k^{\infty}_{min}(y, E) \cong |y - (E - E_{min})| \qquad (18)$$

one has:

$$F(y) = 2\pi \int_{|y|}^{\infty} n_{gr}(k)\, kdk + 2\pi \int_{E_{min}}^{\infty} dE \int_{|y-(E-E_{min})|}^{\infty} P_{ex}(k, E)\, kdk, \qquad (19)$$

or

$$F(y) = f(y) - B(y), \qquad (20)$$

where

$$f(y) = 2\pi \int_{|y|}^{\infty} n(k)\, kdk \qquad (21)$$

is the longitudinal momentum distribution and

$$B(y) = 2\pi \int_{E_{min}}^{\infty} dE \int_{|y|}^{|y-(E-E_{min})|} P_{ex}(k, E)\, kdk \qquad (22)$$

is the contribution arising from $P_{ex}(k, E)$ representing the binding correction to the scaling function.

After taking the derivative of both sides of (20) one gets for the nucleon momentum distribution (NMD):

$$n(k) = -\frac{1}{2\pi y}\left[\frac{dF(y)}{dy} + \frac{dB}{dy}\right], \qquad k = |y|. \qquad (23)$$

Hence, the extraction of the NMD in the y-scaling approach needs the asymptotic scaling function $F(y)$ to be obtained from the experimental data and the binding correction to the NMD dB/dy to be obtained in a realistic way.

The experimental inclusive cross sections σ_2 for ^{3}He [17,18], ^{4}He, ^{12}C and ^{56}Fe [19] and for the nuclear matter [20] give the possibility of determining the experimental scaling function [7,8]:

$$F_1^{\exp}(q, y) = \frac{\sigma_2^{\exp}(q, \omega)}{(Z\sigma_{ep} + N\sigma_{en})} \left| \frac{d\omega}{k\partial \cos\alpha} \right|. \tag{24}$$

The experimental knowledge of the asymptotic scaling function $F(y)$ and the estimate of the binding corrections $B(y)$ for ^{3}He and for complex nuclei [21,22] allow the NMD for ^{2}H, ^{3}He, ^{4}He, ^{12}C, ^{56}Fe and nuclear matter to be obtained [8] using (23). The results for various nuclei confirmed the conclusion that $n(k)$ at $k \le 1$ fm^{-1} can be predicted by the mean field methods, but for $k \ge 2$ fm^{-1} the NMD behaviour depends on correlation effects in nuclei and is almost independent of the mass number A.

In this work we shall apply the NMD obtained in JCM and in the PM accounted for SRC and tensor correlations to calculate the scaling function $F(y)$ and to compare directly with the available experimental data for $F(y)$ in ^{4}He and ^{12}C as well as the binding correction to the scaling function $B(y)$ and the mean kinetic and removal energies in the cases of ^{4}He, ^{12}C, ^{16}O and ^{40}Ca nuclei. The basic relations of the correlation methods used in the calculations are given in the next subsection.

2.2. Phenomenological Correlation Methods

A method to account for the short-range repulsion in the nucleon-nucleon force has been developed by Jastrow [23]. In it the total many-body wave function is written in the form

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_A) = C_A^{-1/2} \prod_{1 \le i \le j \le A} f(\mathbf{r}_{ij})\, \Phi(\mathbf{r}_1, \ldots, \mathbf{r}_A), \tag{25}$$

where A is the number of nucleons with particle coordinates $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A$, Φ is a Slater determinant built up from single-particle wave functions $\varphi_\alpha(\mathbf{r})$ which correspond to the occupied states, and C_A is the normalization constant. The correlation function $f(r_{ij}) = f(|\mathbf{r}_i - \mathbf{r}_j|)$ satisfies the conditions:

$$\begin{aligned} f(r_{ij}) &= 0, \quad \text{for } |\mathbf{r}_i - \mathbf{r}_j| \le r_c, \\ f(r_{ij}) &= 1, \quad \text{for } |\mathbf{r}_i - \mathbf{r}_j| \to \infty, \end{aligned} \tag{26}$$

where r_c is the radius of the nucleon-nucleon repulsive core. The wave function Ψ [eq.(25)] is used as a trial function in varitional calculations of the energy for a system with a given Hamiltonian. Various approximations and appropriate techniques which are based on the variations with respect to the single-particle functions $\varphi_\alpha(\mathrm{r})$ and the correlation function $f(r_{ij})$ have been developed in the JCM.

Gaudin et al. [24] suggested a perturbation expansion method for calculating the one- and two-body density martices. These quantities are written as an expansion in terms of the functions

$$g(r) = |f(r)|^2 - 1, \qquad h(r) = f(r) - 1. \tag{27}$$

Using the lowest-order-cluster approximation, harmonic-oscillator single-particle wave functions and Gaussian form for the function $f(r)$, the nucleon momentum distribution $n(k)$ for ^{4}He has been obtained [11] and compared with the exact Jastrow calculations [25]. An important feature of $n(k)$ is the high-momentum tail at $k > 2$ fm^{-1}. This result shows the role of the Jastrow-type SRC on the high-momentum components of the momentum distribution.

In [12,13] the JCM in its lowest-order approximation (LOA) is applied to calculate short-range correlation effects on the nucleon momentum and density distributions, as well as on the occupation probabilities and natural orbitals in the ^{4}He, ^{16}O and ^{40}Ca nuclei. The obtained analytical expressions for the one-body density matrix, for the momentum and density distributions in [12,13] give a possibility for a detailed study of the quantities sensitive to the SRC.

The short range and tensor correlation effects on the nucleon momentum distributions and form factor have been studied within the phenomenological model from [14,15]. The two-body correlation operator $u(1,2)$ acting on the pair wave function is introduced in the two-body density matrix of the correlated system:

$$\rho(v_1, v_2; v_1', v_2') = \sum_{a,b} [\langle v_1 v_2 | u(1,2) | ab\rangle \langle ab | u^+(1,2) | v_1' v_2'\rangle - \\ - \langle v_1 v_2 | u(1,2) | ab\rangle \langle ab | u^+(1,2) | v_2' v_1'\rangle], \tag{28}$$

where $v \equiv (\mathrm{r}_i, s_i^z, t_i^z)$. In the case of the harmonic-oscillator single-particle wave functions the two-particle state function $|a(1), b(2)\rangle$ is expanded on the basis of the relative and c.m. coordinates, the total angular momentum, and spin and isospin of the pair:

$$|a(1), b(2)\rangle = \sum C_{ab} |nlm\rangle\, |NLM\rangle\, |SS^z\rangle\, |TT^z\rangle, \tag{29}$$

where N, L, M; n, l, m are the radial and angular c.m. and relative motion quantum numbers; S, T, the spin and isospin of the pair; and S^z and T^z, their third components.

The SRC effects are included by means of the operator $u(1, 2)_{s.r.}$ acting on the radial part of the pair wave function

$$[u(1, 2)]_{s.r.} |nlm\rangle = N_{nl}^{-1/2} f(r) |(nlm)\rangle, \tag{30}$$

with

$$f(r) \xrightarrow{r \to 0} 0, \qquad f(r) \xrightarrow{r \to \infty} 1. \tag{31}$$

The tensor correlations are included by using the two-body operator $u(1, 2)_{\text{tens.}}$ that acts both on the angular and the radial parts of the relative motion of the pair. In practical applications the tensor operator is restricted to deuteron-like states only:

$$[u(1, 2)]_{\text{tens.}} |n^3, S_1, J^z, T = 0\rangle = (1 - \eta^2)^{1/2} \varphi_{n0}(r) |n, {}^3S_1, J^z, T = 0\rangle +$$

$$+ \eta \varphi_{n2}(r) |n, {}^3D_1, J^z, T = 0\rangle, \tag{32}$$

where $\varphi_{nl}(r)$ are the radial wave functions, chosen to be the harmonic-oscillator fuctions.

Explicit expressions for the nucleon momentum distributions and form factors are obtained for the ^{4}He, ^{16}O and ^{40}Ca nuclei in [15]. It is shown that the effects of both short-range and tensor correlations lead to the existence of the high-momentum tail of the momentum distribution $n(k)$ which is several orders of magnitude higher than the values of $n(k)$ in the independent-particle models. The tensor correlations are stronger for light nuclei (^{4}He and ^{16}O) than for ^{40}Ca.

The detailed study of the NMD in the JCM [12,13] and in the PM from [14,15] shows that it can be separated into two terms, $n(k) = n_1(k) + n_2(k)$, where the first one ($n_1(k)$) corresponds to the low-momentum region ($k < 2$ fm^{-1}); and the second one ($n_2(k)$), to the high-momentum region ($k \geq 2$ fm^{-1}). This can be related to the conclusion from [8] that the NMD at $k \geq 2$ fm^{-1} is entirely exhausted by $n_{\text{ex}}(k)$. The latter allows us to identify $n_{\text{gr}}(k)$ and $n_{\text{ex}}(k)$ from (7) with $n_1(k)$ and $n_2(k)$, respectively, and to calculate the scaling function, the binding correction to it and the mean kinetic and removal energies within the JCM and the PM for various nuclei.

3. Results of Calculations and Discussion

3.1. Scaling Function and Binding Correction

The effects of the nucleon correlations accounted in the JCM [12,13] and in the PM [14,15] on the y-scaling function $F(y)$ and the binding correction $B(y)$ can be calculated using the momentum distributions obtained in both correlation methods in eqs.(19—22) and the model spectral functions (eqs.(5,6,9)). The theoretical results for the functions $F(y)$ and $B(y)$ in the ^{4}He, ^{12}C, ^{16}O and ^{40}Ca nuclei obtained by using the nucleon momentum distributions from the JCM method [12,13,26] are given in Figs.1—4. We note that: i) At low values of $|y|$ ($|y| \leq 350$ MeV/c) the shape of $F(y)$ for a given nucleus is determined mainly by the first term of eq.(19), which scales in y and is generated by $n_{gr}(k)$. The latter is similar to the momentum distribution predicted in the mean-field approximation; ii) At higher values of $|y|$ ($|y| > 350$ MeV/c) the function $F(y)$ is almost entirely determined by the second term of eq.(19), which is generated by $n_{ex}(k)$, i.e. by the high-momentum part of the momentum distribution. The function $n_{ex}(k)$ is almost independent of the mass number A and contains the effects of the SRC. Concerning the binding correction $B(y)$ to the scaling function we note that it is almost constant at $y \leq -100$ MeV/c and is quite appreciable at

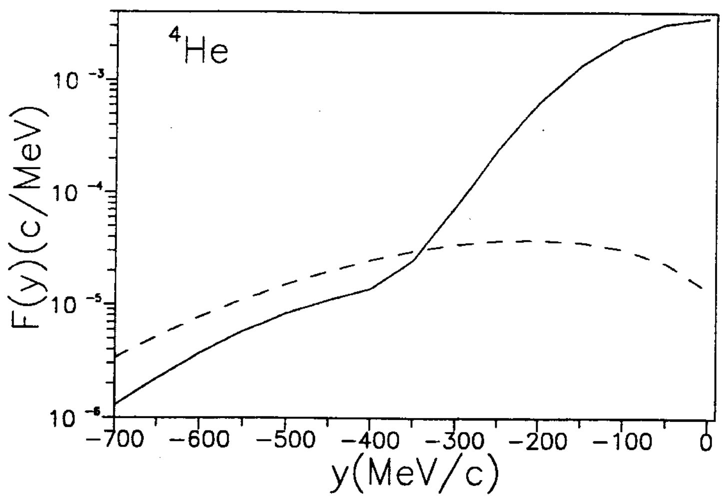

Fig.1. The scaling function $F(y)$ of ^{4}He (solid line) and the function $B(y)$ (dashed line) calculated by using the nucleon momentum distribution $n(k)$ from [13]

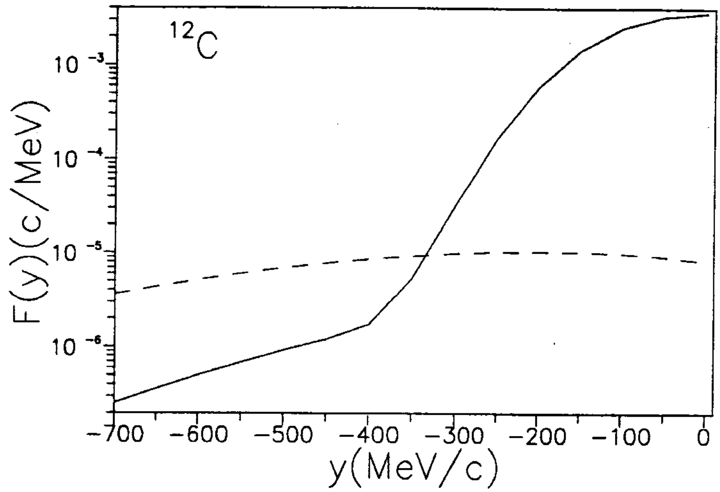

Fig.2. The same as in Fig.1, but for ^{12}C. The nucleon momentum distribution used in the calculation is from [26]

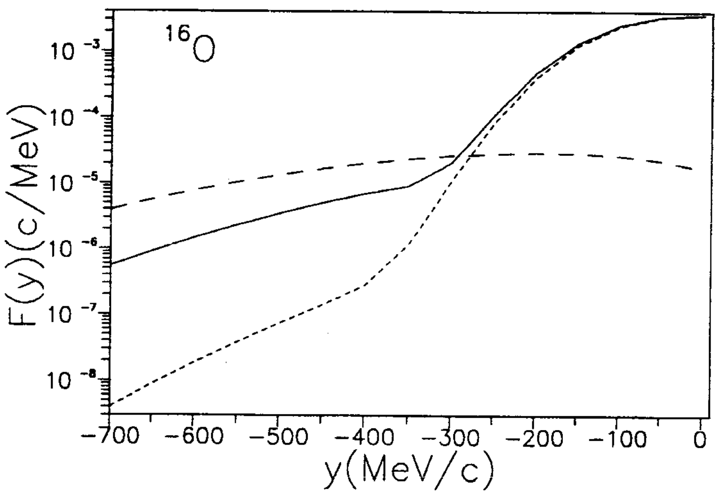

Fig.3.The scaling function $F(y)$ of ^{16}O (solid line) and the function $B(y)$ (long-dashed line) calculated by using the nucleon momentum distribution from [12,13]. The short-dashed line is the scaling function $F(y)$ calculated by using $n(k)$ from [15]

large values of $|y|$. This confirms the conclusions from [8] on the necessity of the electron-nuclei quasielastic cross sections to be calculated in terms of spectral functions and not simply by convoluting the free electron-nucleon cross section with the nucleon momentum distribution.

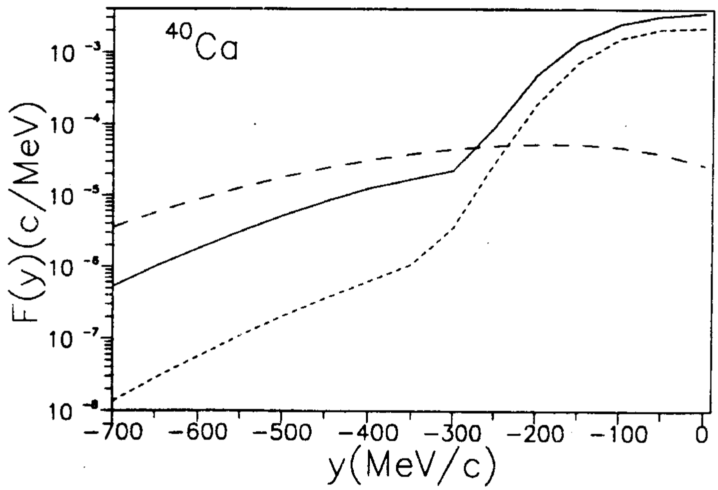

Fig.4. The same as in Fig.3, but for ^{40}Ca

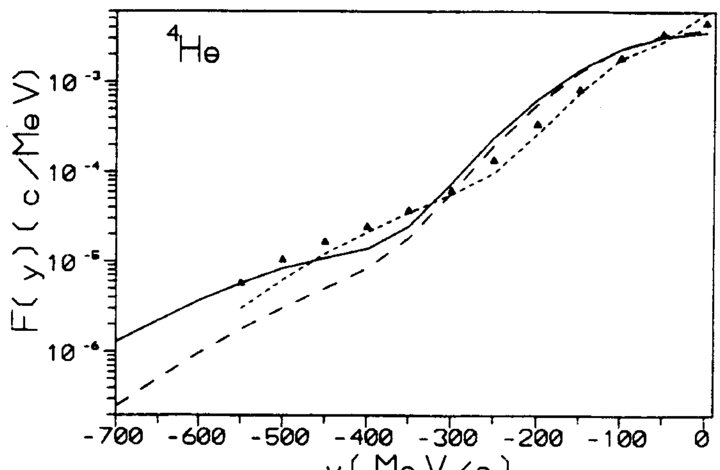

Fig.5. The scaling function $F(y)$ of ^{4}He. The solid triangles represent the experimental scaling function from [10]. The results of this work: calculations by using $n(k)$ from [13] (solid line) and calculations by using $n(k)$ from [15] (long-dashed line). The short-dashed line is the result from [10]

The scaling functions $F(y)$ for the ^{4}He and ^{12}C nuclei calculated in this work are compared in Figs.5 and 6 with the available experimental data for the asymptotic scaling function. They are compared also with the results of our calculations for the scaling function in ^{4}He, ^{16}O and ^{40}Ca (Figs.3—6)

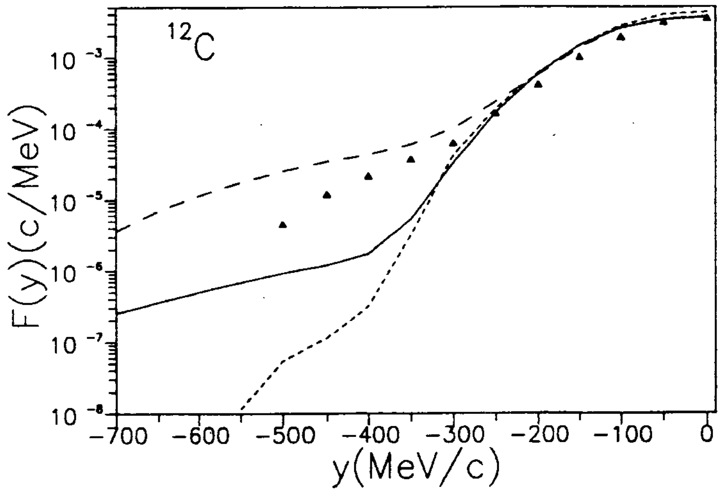

Fig.6. The scaling function $F(y)$ of ^{12}C. The solid triangles represent the experimental scaling function from [10]. The result of this work: calculations by using $n(k)$ from [26] (solid line). The long-dashed line is the result from [27]. The short-dashed line is result in the Hartree-Fock method (taken from [7])

using the nucleon momentum distributions from the phenomenological correlation model accounted for short-range and tensor correlations [15]. In this case $n_{ex}(k)$ is identified with the third terms in the right-hand side of eqs.(10), (11) and (12) in [15] for the ^{4}He, ^{16}O and ^{40}Ca nuclei, correspondingly, while $n_{gr}(k)$ is identified with the sum of all other terms in the right-hand side of the same equations.

In the case of ^{4}He and ^{12}C the scaling functions calculated in [7] and [10] using many-body correlated wave functions from [27] are given in Figs.5 and 6, respectively. The predictions of the Hartree-Fock method (taken from [7]) are shown in Fig.6 for the case of the ^{12}C nucleus.

It can be seen from Fig.5 that the scaling function $F(y)$ for ^{4}He calculated within the JCM is in good agreement with the experimental data and with the results from [10]. The same is true for $F(y)$ calculated using the PM from [15] for $y \geq -350$ MeV/c. The JCM result for $F(y)$ in ^{12}C is similar to that from the correlation method from [7,10,27] for $|y| \leq 300$ MeV/c. The values of $F(y)$ at $|y| > 300$ MeV/c are much larger than those obtained in the Hartree-Fock method. This is due to the nucleon correlation effects in the momentum distribution calculated in the Jastrow method. We note the difference between the results for $F(y)$ at $|y| > 300$ MeV/c obtained in the JCM and in the PM in the cases of ^{16}O and ^{40}Ca. We emphasize the necessity of obtaining experimental data for $F(y)$ in these nuclei as a test of the various correlation methods.

Table. Mean kinetic ($\langle T \rangle$) and mean removal ($\langle E \rangle$) energies, occupation probabilities S_{gr} and S_{ex} calculated within the Hartee-Fock (HF) approximation and in many-body correlation methods [11,12,13,15,28,29,30] and the binding energy per nucleon (E_A/A)

Nuclei		$\langle T \rangle$ MeV	$\langle E \rangle$ MeV	S_{gr}	S_{ex}	E_A/A [eq.(10)] MeV	$(E_A/A)_{exp}$ [32] MeV
^4He	Shell model [28]	17.1	19.8	1.0	0.0	4.20	7.07
	Ref. [28]	21.1	28.2			7.07	
	Ref. [30]	28.7					
	calculated in this work using						
	Ref. [11]	25.96	31.45	0.93	0.07	7.07	
	Ref. [15]	20.35	27.71	0.93	0.07	7.07	
	Ref. [12,13]	25.79	31.34	0.905	0.095	7.07	
^{12}C	HF [29]	17.0	23.0	1.0	0.0	3.77	7.68
	Ref. [29]	37.0	49.0	0.8	0.2	7.68	
	calculated in this work using						
	Ref. [26]	20.97	34.42	0.98	0.02	7.68	
^{16}O	HF [29]	15.0	24.0	1.0	0.0	5.00	7.97
	Ref. [29]	27.0	41.0			7.90	
	Ref. [30]	34.4					
	calculated in this work using						
	Ref. [15]	19.73	34.37	0.94	0.06	7.98	
	Ref. [12,13]	21.28	35.81	0.95	0.05	7.97	
^{40}Ca	HF [29]	16.5	26.6	1.0	0.0	5.26	8.55
	Ref. [29]	36.0	52.1	0.8	0.2	8.51	
	calculated in this work using						
	Ref. [15]	18.71	35.33	0.91	0.09	8.55	
	Ref. [12,13]	23.29	39.80	0.91	0.09	8.55	

3.2. Mean Kinetic and Mean Removal Energy

The study of the nucleon correlation effects using realistic nucleon-nucleon interactions shows [28—30] a substantial increase of the values of the mean kinetic $\langle T\rangle$ and mean removal $\langle E\rangle$ energy with respect to their Hartree-Fock values, as well as a strong relation between the high-momentum components in the momentum distribution. This link is quantitatively explained within the two-nucleon correlation model [31] in which the high-momentum components of a nucleon are generated by its hard interaction with a single nucleon, whereas the remaining $(A-2)$ nucleons (the soft nucleons) move in the mean field with c.m. momentum $k_{A-2} \approx 0$. Eqs. (11—13) allow us to calculate $\langle T\rangle$ and $\langle E\rangle$ using the momentum distributions from [11—15]. We note that $|\varepsilon_A|$, $\langle T\rangle$ and $\langle E\rangle$ have to satisfy the Koltun's sum rule (eq.10)) [16]. The values of the mean kinetic and removal energy, as well as the binding energy per nucleon evaluated by using of eq. (10), which are calculated within the shell model, in the Hartree-Fock method and in various correlation methods for the ^{4}He, ^{12}C, ^{16}O and ^{40}Ca nuclei are given in the Table. It can be seen that the increase of the values of $\langle T\rangle$ and $\langle E\rangle$ due to the correlation effects is quite a general feature of the many-body calculations. The values of $\langle T\rangle$ and $\langle E\rangle$ obtained in our correlation approaches give a correct value for the binding energy per nucleon, which is not the case in the shell model and in the Hartree-Fock method. The values of the occupation probabilities S_{gr} and S_{ex} are listed also in the Table. They are obtained using eqs. (14) and (15) in calculations for ^{4}He, ^{12}C, ^{16}O and ^{40}Ca nuclei. Due to the ground state correlations the values of S_{gr} are less than unity (while $S_{gr}^{HF} = 1$) and the values of S_{ex} are larger than zero (while $S_{ex}^{HF} = 0$).

4. Conclusions

It is shown in the y-scaling method that if only nucleonic degrees of freedom are considered within the framework of the PWIA, then at sufficiently high values of q the structure function $F(q, y)$ becomes a function only of y. The analysis of the y-scaling in the region where it is observed ($y < 0$) allows one to obtain information on the important characteristics of the nucleon dynamics (e.g., on the nucleon momentum distribution) in nuclei. The analysis in the region where the scaling is not observed ($y > 0$) yields information on the effects which break down the impulse approximation and shows the limits of the independent-particle description of the nuclear systems.

The main conclusion of the y-scaling analysis is that the experimentally obtained asymptotic scaling function, as well as the scaling function calculated in various correlation methods differ largely from the scaling function obtained in the MFA. The binding correction to the scaling function can be explicitly evaluated using the nuclear spectral function. These conclusions are confirmed in the present work by the calculations of the scaling functions $F(y)$ and the binding correction $B(y)$ by using realistic momentum distributions obtained within the correlation approaches, such as the Jastrow method [12,13] and the phenomenological correlation model [14,15] in the case of the ^4He, ^{12}C, ^{16}O and ^{40}Ca nuclei. The increased calculated values of the mean kinetic $\langle T \rangle$ and mean removal $\langle E \rangle$ energy in comparison with the shell-model and the Hartree-Fock calculations lead to correct results for the binding energy per nucleon in the nuclei considered. The values of $\langle T \rangle$ and $\langle E \rangle$ obtained in this work can be related to the presence of high-momentum and removal energy components in the many-body spectral function. These values reflect the extent to which the short-range nucleon-nucleon correlations are accounted for in the methods considered in this work.

References

1. Antonov A.N., Hodgson P.E., Petkov I.Zh. — Nucleon Correlations in Nuclei, Springer—Verlag, Berlin—Heidelberg—New-York, 1993.
2. West G.B. — Phys. Rep., 1975, 18, p.263.
3. Ciofi degli Atti C., Pace E., Salme G. — Few Body Systems, Suppl., 1986, 1, p.280.
4. Ciofi degli Atti C., Pace E., Salme G. — Phys. Rev., 1987, C36, p.1208.
5. Ciofi degli Atti C. — Nucl. Phys., 1987, A463, p.127c.
6. Ciofi degli Atti C., Pace E., Salme G. — Phys. Rev., 1989, C39, p.259.
7. Ciofi degli Atti C., Pace E., Salme G. — Nucl. Phys., 1989, A497, p.361c.
8. Ciofi degli Atti C., Pace E., Salme G. — Preprint INFN-ISS 90/8, Roma, 1990.
9. Ciofi degli Atti C., Pace E., Salme G. — Preprint INFN-ISS 87/7, Roma, 1987: Nucl. Phys., 1990, A508, p.349c.
10. Ciofi degli Atti C., Pace E., Salme G. — Phys. Rev., 1991, C43, p.1155.
11. Bohigas O., Stringari S. — Phys. Lett., 1980, 95, p.9.
12. Stoitsov M.V., Antonov A.N., Dimitrova S.S. — Phys. Rev., 1993, C47, p.R455; Phys. Rev., 1993, C48, p.74.
13. Stoitsov M.V., Antonov A.N. — Z. Phys., 1993, A345, p.259; Stoitsov M.V., Antonov A.N., Dimitrova S.S. — Z. Phys., 1993, A345, p.359.

14. Dellagiacoma F., Orlandini G., Traini M. — Nucl. Phys., 1983, A393, p.95.
15. Traini M., Orlandini G. — Z. Phys., 1985, A321, p.479.
16. Koltun D.S. — Phys. Rev. Lett., 1972, 28, p.182; Phys. Rev., 1974, C9, p.484.
17. Day D.B. et al. — Phys. Rev. Lett., 1979, 43, p.1143.
18. Rock S. et al. — Phys. Rev., 1982, C26, p.1592.
19. Day D.B. et al. — Phys. Rev. Lett., 1987, 59, p.427.
20. Day D.B. et al. — Phys. Rev., 1989, C40, p.1011.
21. Ciofi degli Atti C. et al. — In: Proceedings of the 4th Workshop on Perspectives in Nuclear Physics at Intermediate Energies, Trieste, 8—12 May 1989, Ed. S.Boffi, C.Ciofi degli Atti, Giannini M., World Scientific Publ. Co., Singapore, 1989, p.312.
22. Ciofi degli Atti C., Liuti S., Simula S. — Phys. Rev., 1990, C41, p.R2474.
23. Jastrow R. — Phys. Rev., 1955, 98, p.1479.
24. Gaudin M., Gillespie J., Pipka G. — Nucl. Phys., 1971, A176, p.237.
25. Dal Ri M., Stringari S., Bohigas O. — Nucl. Phys., 1982, A376, p.81.
26. Stoitsov M.V., Anotonov A.N., Dimitrova S.S. — In: Proceedings of the 6th Workshop on Perspectives in Nuclear Physics at Intermediate Energies, Trieste 3—7 May 1993, Ed. S.Boffi, C.Ciofi degli Atti, M.Giannini, World Scientific Publ. Co., Singapore (in print).
27. Benhar O. et al. — Phys. Lett., 1986, 177B, p.135.
28. Ciofi degli Atti C., Liuti S. — Phys. Rev., 1990, C41, p.1100.
29. Ciofi degli Atti C., Liuti S. — Phys. Rev., 1989, 225B, p.215.
30. Pieper S.C., Wiringa R.B., Pandharipande V.R. — Phys. Rev., 1992, C46, p.1741.
31. Frankfurt L., Strikman M. — Phys. Rep., 1981, 76, p.215; Phys. Rep., 1988, 160, p.236.
32. Kravtsov V.A. — Atomic Masses and Nuclear Binding Energies, Moscow, Atomizdat, 1974.

Received on June 28, 1994.